> data<- read.csv("D:/Users/marin/Documents/podaci.csv")

> attach(data)

> # Define variables

> X <- cbind(Ag, Mining, Constr, Manuf, Manuf\_nd, Transp, Comm, Energy, TradeW, TradeR,

+ RE, Services, Govt)

> X

Ag Mining Constr Manuf Manuf\_nd Transp Comm Energy TradeW TradeR RE

[1,] 2.0 1.5 4.2 10.5 11.8 2.9 2.9 3.6 6.3 9.9 12.8

[2,] 1.5 22.4 4.1 1.1 3.7 12.1 2.0 1.5 2.9 6.5 10.7

[3,] 1.7 1.3 5.8 11.5 3.0 2.8 2.2 2.7 6.3 10.5 18.9

[4,] 5.1 1.0 4.0 12.8 11.8 4.4 2.4 4.2 6.1 10.2 11.4

[5,] 2.1 0.6 3.3 9.0 5.0 2.6 2.5 1.8 6.8 8.9 22.7

[6,] 1.8 1.7 5.4 7.7 4.5 3.3 5.7 2.2 6.3 9.7 17.0

[7,] 0.7 0.0 3.3 11.0 5.7 1.8 2.3 2.2 6.6 7.4 28.2

[8,] 1.0 0.0 3.4 4.5 16.6 1.6 1.3 2.4 4.0 6.0 35.4

[9,] 1.8 0.2 4.7 4.6 3.5 3.1 3.0 2.8 7.3 11.2 21.8

[10,] 1.8 0.4 3.9 7.4 10.7 4.0 4.5 2.7 8.8 8.9 16.4

[11,] 1.2 0.1 4.8 0.8 2.3 4.5 3.1 2.7 4.0 11.5 21.4

[12,] 6.3 0.6 5.9 15.0 5.6 3.5 1.6 3.7 6.1 9.9 12.3

[13,] 1.4 0.3 4.2 11.3 7.9 3.8 2.3 3.1 7.7 8.1 19.2

[14,] 1.8 0.5 4.6 21.4 10.3 3.5 1.4 3.1 6.0 9.1 13.1

[15,] 7.6 0.2 4.1 13.2 10.8 3.3 2.0 2.8 6.8 8.3 14.3

[16,] 4.4 1.4 4.2 10.4 7.9 3.9 3.6 3.4 7.8 9.6 12.7

[17,] 2.6 2.6 3.9 14.9 13.2 3.9 1.5 2.9 5.8 8.9 11.2

[18,] 1.2 14.8 4.2 3.7 15.3 3.3 1.9 3.6 5.3 7.8 12.1

[19,] 1.8 0.1 4.5 7.9 10.6 2.3 2.0 3.1 6.0 11.1 18.5

[20,] 0.9 0.1 5.0 4.1 4.5 2.1 2.9 2.9 6.3 8.7 21.4

[21,] 0.6 0.1 3.2 10.5 4.9 2.0 1.9 2.5 7.1 7.8 24.4

[22,] 1.0 0.4 3.8 20.0 7.2 2.4 1.6 2.7 7.2 8.9 15.8

[23,] 2.9 0.6 4.4 11.0 8.2 3.7 1.9 2.1 8.3 8.7 17.9

[24,] 3.2 0.9 3.9 13.0 10.5 3.2 2.4 5.1 5.6 10.0 11.5

[25,] 1.8 0.4 4.6 11.3 10.1 4.0 3.5 2.8 7.3 9.1 14.7

[26,] 5.1 4.9 4.8 4.4 3.3 5.4 2.1 5.0 6.4 9.9 13.4

[27,] 9.2 0.2 4.4 7.0 7.1 6.8 2.1 1.4 7.4 8.3 14.8

[28,] 0.8 3.7 8.4 3.1 1.7 2.9 1.8 3.0 4.6 9.4 18.4

[29,] 0.7 0.1 3.5 16.4 5.7 1.5 1.7 4.4 6.2 9.1 22.2

[30,] 0.6 0.0 3.5 4.3 9.4 3.2 4.1 2.7 9.1 7.3 23.2

[31,] 1.9 7.1 4.6 14.4 2.1 2.5 1.7 3.5 4.3 8.9 13.9

[32,] 0.5 0.1 2.9 5.9 5.9 2.1 3.6 2.4 6.2 6.9 29.7

[33,] 2.3 0.1 4.2 10.2 16.7 2.8 2.0 3.0 6.4 8.9 14.6

[34,] 10.6 3.1 4.9 4.6 2.9 4.9 1.9 4.0 8.8 9.1 12.7

[35,] 1.1 0.4 3.9 17.9 9.3 2.9 1.9 3.0 7.1 9.2 15.3

[36,] 2.1 5.3 3.2 10.2 7.1 3.7 2.8 3.5 6.1 10.0 12.5

[37,] 3.0 0.1 5.4 16.0 4.5 3.4 1.9 2.5 8.0 8.7 16.3

[38,] 1.0 0.5 3.8 11.3 9.4 3.0 2.2 3.6 5.9 8.6 19.0

[39,] 0.8 0.1 3.5 11.1 5.6 1.5 2.5 3.1 5.6 8.7 22.6

[40,] 1.4 0.2 4.7 10.4 16.1 2.4 2.0 3.6 5.8 10.3 13.3

[41,] 9.9 1.2 3.7 11.1 3.4 3.1 1.6 3.1 6.1 8.8 20.6

[42,] 1.2 0.3 3.9 12.4 10.5 4.8 2.0 1.0 7.4 10.9 13.8

[43,] 1.3 7.5 4.4 8.6 7.7 3.7 3.2 3.7 7.3 8.7 14.3

[44,] 1.2 3.2 5.7 9.1 4.9 4.1 2.0 2.6 6.1 10.3 16.5

[45,] 2.3 0.3 4.3 12.3 5.8 2.2 2.6 3.6 6.1 9.7 17.8

[46,] 1.0 0.5 4.4 6.0 9.2 2.8 3.4 2.4 5.6 8.2 17.8

[47,] 2.9 0.2 4.8 8.8 4.4 3.2 3.2 1.6 7.4 9.7 18.3

[48,] 0.6 8.0 4.6 7.2 10.9 3.3 2.3 7.5 5.3 8.7 11.2

[49,] 2.2 0.3 4.3 16.5 11.3 3.2 1.5 2.4 6.3 8.6 16.1

[50,] 2.1 31.6 3.7 1.4 4.3 6.4 1.4 6.4 3.2 6.5 10.4

Services Govt

[1,] 16.1 15.5

[2,] 11.9 19.6

[3,] 20.2 13.0

[4,] 14.8 11.8

[5,] 23.1 11.5

[6,] 21.6 13.1

[7,] 21.8 9.0

[8,] 14.3 9.4

[9,] 23.4 12.4

[10,] 18.0 12.5

[11,] 22.2 21.3

[12,] 16.3 13.2

[13,] 20.7 10.0

[14,] 15.3 9.8

[15,] 15.3 11.4

[16,] 16.7 14.1

[17,] 15.0 13.6

[18,] 15.7 10.9

[19,] 18.7 13.5

[20,] 23.2 17.8

[21,] 25.8 9.2

[22,] 18.5 10.5

[23,] 19.5 10.9

[24,] 16.0 14.9

[25,] 19.1 11.2

[26,] 19.2 15.9

[27,] 17.1 14.1

[28,] 32.3 9.9

[29,] 19.4 9.1

[30,] 21.8 10.8

[31,] 17.5 17.7

[32,] 22.9 11.1

[33,] 15.4 13.2

[34,] 17.5 15.1

[35,] 17.7 10.5

[36,] 17.4 16.2

[37,] 18.3 11.9

[38,] 21.2 10.5

[39,] 22.7 12.1

[40,] 15.1 14.8

[41,] 15.6 11.7

[42,] 19.6 12.1

[43,] 18.0 11.7

[44,] 19.6 14.7

[45,] 20.8 12.1

[46,] 19.9 18.9

[47,] 20.7 14.9

[48,] 16.5 13.8

[49,] 16.6 10.9

[50,] 9.6 12.9

> # Descriptive statistics

> summary(X)

Ag Mining Constr Manuf

Min. : 0.500 Min. : 0.000 Min. :2.900 Min. : 0.800

1st Qu.: 1.025 1st Qu.: 0.200 1st Qu.:3.825 1st Qu.: 6.250

Median : 1.800 Median : 0.450 Median :4.200 Median :10.400

Mean : 2.480 Mean : 2.624 Mean :4.338 Mean : 9.784

3rd Qu.: 2.525 3rd Qu.: 1.650 3rd Qu.:4.675 3rd Qu.:12.375

Max. :10.600 Max. :31.600 Max. :8.400 Max. :21.400

Manuf\_nd Transp Comm Energy

Min. : 1.700 Min. : 1.500 Min. :1.300 Min. :1.000

1st Qu.: 4.500 1st Qu.: 2.650 1st Qu.:1.900 1st Qu.:2.500

Median : 7.150 Median : 3.200 Median :2.100 Median :2.950

Mean : 7.696 Mean : 3.476 Mean :2.398 Mean :3.112

3rd Qu.:10.500 3rd Qu.: 3.875 3rd Qu.:2.875 3rd Qu.:3.600

Max. :16.700 Max. :12.100 Max. :5.700 Max. :7.500

TradeW TradeR RE Services

Min. :2.900 Min. : 6.000 Min. :10.40 Min. : 9.60

1st Qu.:5.825 1st Qu.: 8.600 1st Qu.:13.15 1st Qu.:16.15

Median :6.300 Median : 8.900 Median :16.20 Median :18.40

Mean :6.348 Mean : 9.002 Mean :17.09 Mean :18.71

3rd Qu.:7.275 3rd Qu.: 9.850 3rd Qu.:19.15 3rd Qu.:20.77

Max. :9.100 Max. :11.500 Max. :35.40 Max. :32.30

Govt

Min. : 9.00

1st Qu.:10.90

Median :12.25

Mean :12.93

3rd Qu.:14.55

Max. :21.30

> cor(X)

Ag Mining Constr Manuf Manuf\_nd

Ag 1.00000000 -0.06446456 0.08498040 0.03208436 -0.14533029

Mining -0.06446456 1.00000000 -0.02146761 -0.42367130 -0.13794386

Constr 0.08498040 -0.02146761 1.00000000 -0.12993364 -0.31780734

Manuf 0.03208436 -0.42367130 -0.12993364 1.00000000 0.20372851

Manuf\_nd -0.14533029 -0.13794386 -0.31780734 0.20372851 1.00000000

Transp 0.27917762 0.61153483 0.07516000 -0.35693665 -0.17641840

Comm -0.18418380 -0.19271386 -0.02310018 -0.31738049 -0.09988080

Energy 0.04325752 0.39044648 0.01300031 -0.05083012 0.07091362

TradeW 0.24539204 -0.55305518 -0.08691544 0.27073259 0.03900687

TradeR 0.09464267 -0.39599385 0.40113015 0.19462642 -0.12082255

RE -0.30129373 -0.40633100 -0.25294548 -0.18205552 -0.13333291

Services -0.32191590 -0.45971557 0.32364308 -0.15904326 -0.45771026

Govt 0.11033873 0.23067866 0.18104989 -0.41051203 -0.23707649

Transp Comm Energy TradeW TradeR

Ag 0.27917762 -0.18418380 0.04325752 0.24539204 0.09464267

Mining 0.61153483 -0.19271386 0.39044648 -0.55305518 -0.39599385

Constr 0.07516000 -0.02310018 0.01300031 -0.08691544 0.40113015

Manuf -0.35693665 -0.31738049 -0.05083012 0.27073259 0.19462642

Manuf\_nd -0.17641840 -0.09988080 0.07091362 0.03900687 -0.12082255

Transp 1.00000000 -0.04911955 -0.05572106 -0.21353750 -0.14778210

Comm -0.04911955 1.00000000 -0.16859364 0.33018068 0.12467627

Energy -0.05572106 -0.16859364 1.00000000 -0.26707721 0.02953892

TradeW -0.21353750 0.33018068 -0.26707721 1.00000000 0.16636809

TradeR -0.14778210 0.12467627 0.02953892 0.16636809 1.00000000

RE -0.50348731 0.11971090 -0.37884901 0.04049752 -0.30862993

Services -0.42168031 0.30914590 -0.31383479 0.23908339 0.20216576

Govt 0.42750613 0.19333215 0.04539830 -0.34283068 0.28658841

RE Services Govt

Ag -0.30129373 -0.3219159 0.1103387

Mining -0.40633100 -0.4597156 0.2306787

Constr -0.25294548 0.3236431 0.1810499

Manuf -0.18205552 -0.1590433 -0.4105120

Manuf\_nd -0.13333291 -0.4577103 -0.2370765

Transp -0.50348731 -0.4216803 0.4275061

Comm 0.11971090 0.3091459 0.1933322

Energy -0.37884901 -0.3138348 0.0453983

TradeW 0.04049752 0.2390834 -0.3428307

TradeR -0.30862993 0.2021658 0.2865884

RE 1.00000000 0.5192887 -0.3506442

Services 0.51928874 1.0000000 -0.1795611

Govt -0.35064424 -0.1795611 1.0000000

> # Principal component analysis

> pca1 <- princomp(X, scores=TRUE, cor=TRUE)

> summary(pca1)

Importance of components:

Comp.1 Comp.2 Comp.3 Comp.4 Comp.5

Standard deviation 1.7987525 1.4954801 1.3999420 1.1663403 1.07583525

Proportion of Variance 0.2488854 0.1720354 0.1507567 0.1046423 0.08903242

Cumulative Proportion 0.2488854 0.4209209 0.5716776 0.6763199 0.76535232

Comp.6 Comp.7 Comp.8 Comp.9 Comp.10

Standard deviation 0.93184458 0.85116719 0.78471605 0.5641253 0.4851322

Proportion of Variance 0.06679495 0.05572966 0.04736764 0.0244798 0.0181041

Cumulative Proportion 0.83214726 0.88787692 0.93524456 0.9597244 0.9778285

Comp.11 Comp.12 Comp.13

Standard deviation 0.38943836 0.36945813 8.279806e-03

Proportion of Variance 0.01166633 0.01049995 5.273476e-06

Cumulative Proportion 0.98949478 0.99999473 1.000000e+00

> # Loadings of principal components

> loadings(pca1)

Loadings:

Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9 Comp.10

Ag 0.135 -0.385 -0.373 0.411 0.245 -0.433 -0.277 0.152 -0.217

Mining 0.470 0.260 0.164 0.276 0.148 -0.116

Constr 0.393 -0.257 0.350 0.196 0.370 -0.499 0.371

Manuf -0.183 -0.376 -0.375 0.147 0.111 -0.198 0.151 0.500 0.387

Manuf\_nd -0.459 -0.465 -0.217 0.102 -0.592 -0.102

Transp 0.418 0.147 -0.365 0.143 -0.169 0.302 -0.264 -0.108

Comm -0.152 0.316 -0.343 -0.550 0.267 0.145 0.101 0.436 -0.383

Energy 0.247 -0.138 0.416 -0.202 0.689 -0.199 0.116

TradeW -0.315 -0.290 -0.442 0.353 0.254 -0.253 0.455

TradeR 0.261 -0.507 0.227 -0.252 -0.143 -0.146 0.100 -0.515 -0.432

RE -0.363 0.447 0.173 -0.359 -0.106 -0.169

Services -0.380 0.384 0.127 0.183 0.125 0.101 0.133 -0.222 0.204

Govt 0.289 0.369 -0.295 -0.306 -0.428 0.121 0.171 0.548

Comp.11 Comp.12 Comp.13

Ag -0.286 0.206

Mining -0.475 -0.282 0.500

Constr -0.141 0.256

Manuf 0.138 0.406

Manuf\_nd 0.142 -0.122 0.338

Transp 0.507 0.407 0.144

Comm

Energy 0.282 0.286

TradeW -0.322 0.210 0.112

TradeR -0.181 0.106

RE -0.127 0.496 0.451

Services 0.458 -0.460 0.320

Govt 0.238

Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9

SS loadings 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000

Proportion Var 0.077 0.077 0.077 0.077 0.077 0.077 0.077 0.077 0.077

Cumulative Var 0.077 0.154 0.231 0.308 0.385 0.462 0.538 0.615 0.692

Comp.10 Comp.11 Comp.12 Comp.13

SS loadings 1.000 1.000 1.000 1.000

Proportion Var 0.077 0.077 0.077 0.077

Cumulative Var 0.769 0.846 0.923 1.000

> #pca1$loadings

> # Scree plot of eigenvalues

> plot(pca1)

> screeplot(pca1, type="line", main="Scree Plot")



>

> # Biplot of score variables

> biplot(pca1)



> # Scores of the components

> pca1$scores[1:10,]

Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6

[1,] 0.4896329 -0.2840153 -0.92052695 0.08785947 -1.7536053 -0.25729127

[2,] 6.6852235 1.5422261 2.72534093 -2.09383081 0.9621524 -2.42578094

[3,] -0.7517456 1.4884587 -0.86440944 1.23947821 0.7013320 -0.40238064

[4,] 1.0000343 -1.2527044 -1.79705334 -0.15952525 -0.6286758 0.31143230

[5,] -1.8143951 0.3083173 1.07282748 -0.62096975 0.5873437 -0.19317967

[6,] -1.1231352 2.7966578 0.12662810 -0.83439457 -1.5916798 0.58129969

[7,] -2.4543289 -0.8838887 2.26111646 -0.17696795 1.0153673 0.09693667

[8,] -0.9918322 -2.9538499 4.02639673 0.22036149 0.4781388 -1.24008960

[9,] -1.5476850 2.2030680 -0.14284640 0.17108744 -0.1473670 0.46797314

[10,] -0.9920176 0.2998176 -0.05468377 -2.18146390 -1.8755534 0.87740143

Comp.7 Comp.8 Comp.9 Comp.10 Comp.11 Comp.12

[1,] -0.3122104 -0.1332644 0.16656810 0.12841084 -0.14054526 -0.11795741

[2,] 1.2416981 0.8350926 -0.02638485 -0.03282346 0.60577408 0.42502678

[3,] 0.2571765 0.1730944 0.13204988 -0.33229286 -0.67623158 0.53472823

[4,] -0.2804304 -0.2515513 -0.26955035 -0.91416815 0.62811995 0.03117104

[5,] -0.3766092 0.6847873 -0.60181782 -0.15025628 -0.05043902 -0.57785734

[6,] 1.0428510 0.1619501 1.59879806 -1.52538907 -0.02066718 -0.20311745

[7,] -0.2158679 0.5350483 0.06689563 -0.09648740 -0.09564494 0.11018733

[8,] -1.5630880 -2.3471516 0.38662079 -0.46928579 -0.24839536 0.84153331

[9,] -0.1884953 0.0521831 -1.26973408 -0.64521367 -0.42080320 0.29586008

[10,] 0.9428300 -0.1629925 0.11552867 -0.08515134 -0.14312824 0.31799226

Comp.13

[1,] -0.0022081279

[2,] 0.0038008406

[3,] -0.0060317043

[4,] 0.0028593823

[5,] -0.0072707745

[6,] -0.0025178811

[7,] 0.0001849203

[8,] -0.0004708541

[9,] -0.0079586042

[10,] -0.0020137002

> # Rotation

> #varimax(pca1$rotation)

> #promax(pca1$rotation)

> # Factor analysis - different results from other softwares and no rotation

> fa1 <- factanal(X, factor=3)

> fa1

Call:

factanal(x = X, factors = 3)

Uniquenesses:

Ag Mining Constr Manuf Manuf\_nd Transp Comm Energy

0.863 0.005 0.550 0.518 0.533 0.536 0.874 0.782

TradeW TradeR RE Services Govt

0.653 0.420 0.005 0.284 0.755

Loadings:

Factor1 Factor2 Factor3

Ag 0.154 0.335

Mining 0.825 -0.551 -0.108

Constr 0.116 0.656

Manuf -0.117 0.641 -0.238

Manuf\_nd 0.401 -0.552

Transp 0.659 -0.144

Comm -0.198 0.294

Energy 0.461

TradeW -0.344 0.471

TradeR 0.527 0.547

RE -0.847 -0.498 -0.171

Services -0.613 -0.191 0.551

Govt 0.337 0.363

Factor1 Factor2 Factor3

SS loadings 2.747 1.799 1.674

Proportion Var 0.211 0.138 0.129

Cumulative Var 0.211 0.350 0.479

Test of the hypothesis that 3 factors are sufficient.

The chi square statistic is 393.94 on 42 degrees of freedom.

The p-value is 1.01e-58

> fa2 <- factanal(X, factor=3, rotation="varimax")

> fa2

Call:

factanal(x = X, factors = 3, rotation = "varimax")

Uniquenesses:

Ag Mining Constr Manuf Manuf\_nd Transp Comm Energy

0.863 0.005 0.550 0.518 0.533 0.536 0.874 0.782

TradeW TradeR RE Services Govt

0.653 0.420 0.005 0.284 0.755

Loadings:

Factor1 Factor2 Factor3

Ag 0.154 0.335

Mining 0.825 -0.551 -0.108

Constr 0.116 0.656

Manuf -0.117 0.641 -0.238

Manuf\_nd 0.401 -0.552

Transp 0.659 -0.144

Comm -0.198 0.294

Energy 0.461

TradeW -0.344 0.471

TradeR 0.527 0.547

RE -0.847 -0.498 -0.171

Services -0.613 -0.191 0.551

Govt 0.337 0.363

Factor1 Factor2 Factor3

SS loadings 2.747 1.799 1.674

Proportion Var 0.211 0.138 0.129

Cumulative Var 0.211 0.350 0.479

Test of the hypothesis that 3 factors are sufficient.

The chi square statistic is 393.94 on 42 degrees of freedom.

The p-value is 1.01e-58

> fa3 <- factanal(X, factors=3, rotation="varimax", scores="regression")

> fa3

Call:

factanal(x = X, factors = 3, scores = "regression", rotation = "varimax")

Uniquenesses:

Ag Mining Constr Manuf Manuf\_nd Transp Comm Energy

0.863 0.005 0.550 0.518 0.533 0.536 0.874 0.782

TradeW TradeR RE Services Govt

0.653 0.420 0.005 0.284 0.755

Loadings:

Factor1 Factor2 Factor3

Ag 0.154 0.335

Mining 0.825 -0.551 -0.108

Constr 0.116 0.656

Manuf -0.117 0.641 -0.238

Manuf\_nd 0.401 -0.552

Transp 0.659 -0.144

Comm -0.198 0.294

Energy 0.461

TradeW -0.344 0.471

TradeR 0.527 0.547

RE -0.847 -0.498 -0.171

Services -0.613 -0.191 0.551

Govt 0.337 0.363

Factor1 Factor2 Factor3

SS loadings 2.747 1.799 1.674

Proportion Var 0.211 0.138 0.129

Cumulative Var 0.211 0.350 0.479

Test of the hypothesis that 3 factors are sufficient.

The chi square statistic is 393.94 on 42 degrees of freedom.

The p-value is 1.01e-58

> dat = cor(X) # correlation matrix

>

> # Maximum Likelihood Factor Analysis without varimax rotation factanal performs

> mlm = factanal(da, 3, rotation = "none", covmat = dat)

> load = mlm$loadings # estimated factor loadings

> ld = cbind(load[, 1], load[, 2], load[, 3]) # the estimated factor loadings matrix

> com = diag(ld %\*% t(ld)) # communalities are calculated

> psi = diag(dat) - diag(ld %\*% t(ld)) # specific variances are calculated

> tbl = cbind(load[, 1], load[, 2], load[, 3], com, psi)

>

> dev.new()

> par(mfcol = c(2, 2))

>

> # plot first factor against second

> plot(load[, 1], load[, 2], type = "n", xlab = "x", ylab = "y", main = "Factors21 - theta21",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4, ylim = c(-0.6, 0.6))

> text(load[, 1], load[, 2], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # plot first factor against third

> plot(load[, 1], load[, 3], type = "n", xlab = "x", ylab = "y", main = "Factors31 - theta31",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4, ylim = c(-0.4, 0.4))

> text(load[, 1], load[, 3], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # plot second factor against third

> plot(load[, 2], load[, 3], type = "n", xlab = "x", ylab = "y", main = "Factors32 - theta32",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4, xlim = c(-0.6, 0.6),

+ ylim = c(-0.4, 0.4))

> text(load[, 2], load[, 3], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # Maximum Likelihood Factor Analysis after varimax rotation

> var = varimax(ld) # rotates the factor loadings matrix

> load = var$loadings # estimated factor loadings after varimax

> vl = cbind(load[, 1], load[, 2], load[, 3])

> com = diag(vl %\*% t(vl)) # communalities are calculated

> psi = diag(dat) - diag(vl %\*% t(vl)) # specific variances are calculated

> tbl = cbind(load[, 1], load[, 2], load[, 3], com, psi)

>

> dev.new()

> par(mfcol = c(2, 2))

>

> # plot first factor against second

> plot(load[, 1], load[, 2], type = "n", xlab = "x", ylab = "y", main = "Factors21 - theta21",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4, xlim = c(-1, 1))

> text(load[, 1], load[, 2], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # plot first factor against third

> plot(load[, 1], load[, 3], type = "n", xlab = "x", ylab = "y", main = "Factors31 - theta31",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4, xlim = c(-1, 1))

> text(load[, 1], load[, 3], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # plot second factor against third

> plot(load[, 2], load[, 3], type = "n", xlab = "x", ylab = "y", main = "Factors32 - theta32",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4, xlim = c(-1, 1))

> text(load[, 2], load[, 3], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # Principal Component Method after varimax rotation spectral decomposition

> e = eigen(dat)

> eigval = e$values[1:3]

> eigvec = e$vectors[, 1:3]

> E = matrix(eigval, nrow(dat), ncol = 3, byrow = T)

> Q = sqrt(E) \* eigvec # the estimated factor loadings matrix

> pcm = varimax(Q) # rotates the factor loadings matrix

> load = pcm$loadings # estimated factor loadings after varimax

> ld = cbind(load[, 1], load[, 2], load[, 3])

> com = diag(ld %\*% t(ld)) # communalities are calculated

> psi = diag(dat) - diag(ld %\*% t(ld)) # specific variances are calculated

> tbl = cbind(load[, 1], load[, 2], load[, 3], com, psi)

>

> dev.new()

> par(mfcol = c(2, 2))

>

> # plot first factor against second

> plot(load[, 1], load[, 2], type = "n", xlab = "x", ylab = "y", main = "Factors21 - theta21",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4)

> text(load[, 1], load[, 2], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # plot first factor against third

> plot(load[, 1], load[, 3], type = "n", xlab = "x", ylab = "y", main = "Factors31 - theta31",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4)

> text(load[, 1], load[, 3], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # plot second factor against third

> plot(load[, 2], load[, 3], type = "n", xlab = "x", ylab = "y", main = "Factors32 - theta32",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4)

> text(load[, 2], load[, 3], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # Principal Factor Method after varimax rotation inverse of the correlation matrix

> f = solve(dat)

> psiini = diag(1/f[row(f) == col(f)]) # preliminary estimate of psi

> psi = psiini

> for (i in 1:10) {

+ ee = eigen(dat - psi)

+ eigval = ee$values[1:3]

+ eigvec = ee$vectors[, 1:3]

+ EE = matrix(eigval, nrow(dat), ncol = 3, byrow = T)

+ QQ = sqrt(EE) \* eigvec

+ psiold = psi

+ psi = diag(as.vector(1 - t(colSums(t(QQ \* QQ)))))

+ i = i + 1

+ z = psi - psiold

+ convergence = z[row(z) == col(z)]

+ }

> pfm = varimax(QQ) # rotates the factor loadings matrix

> load = pfm$loadings # estimated factor loadings after varimax

> ld = cbind(load[, 1], load[, 2], load[, 3])

> com = diag(ld %\*% t(ld)) # communalities are calculated

> psi = diag(dat) - diag(ld %\*% t(ld)) # specific variances are calculated

> tbl = cbind(load[, 1], load[, 2], load[, 3], com, psi)

>

> dev.new()

> par(mfcol = c(2, 2))

>

> # plot first factor against second

> plot(load[, 1], load[, 2], type = "n", xlab = "x", ylab = "y", main = "Factors21 - theta21",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4)

> text(load[, 1], load[, 2], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # plot first factor against third

> plot(load[, 1], load[, 3], type = "n", xlab = "x", ylab = "y", main = "Factors31 - theta31",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4, ylim = c(-1, 1))

> text(load[, 1], load[, 3], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)

>

> # plot second factor against third

> plot(load[, 2], load[, 3], type = "n", xlab = "x", ylab = "y", main = "Factors32 - theta32",

+ font.main = 1, cex.lab = 1.1, cex.axis = 1.1, cex.main = 1.4, ylim = c(-1, 1))

> text(load[, 2], load[, 3], colnames(data), cex = 1.1)

> abline(h = 0, v = 0)







